Phase shifts of parallel currents in a single-layer model of superconducting cable

Fedor Gömöry, Lubomir Frolek, Ján Šouc, Francesco Grilli

***Abstract*—To reach large current carrying capability and mechanical flexibility, superconducting cables consist of many single conductors connected in parallel. Distribution of AC current into these parallel paths is controlled by their impedances, which often are not identical. We have found that, even in the case of a cable model made from straight Bi-2223 tapes placed in just one single layer, the non-uniformity of contact resistances causes a dispersion in the amplitudes and phases of AC's in individual tapes. Two models, one analytical and other a Finite Element Model was used to predict the influence of phase shifts on AC loss and its measurement by electrical method. Due to phase shifts of local magnetic fields, the purely inductive component of voltage taken by an arbitrary pair of taps is not exactly out-of-phase regarding the total cable current. Then, its product with the cable current is evaluated as a false loss-contributing component. Fortunately, with the use of voltage taps and wires regularly distributed around the cable, true value of AC loss can be recovered.**

***Index Terms*— superconducting transmission lines, electrical variables measurement, loss measurement, power cable testing**

I. Introduction

IN the stage of development of a superconducting power transmission cable, experimental tests of AC loss are commonly provided on a short (~1m) models. The sensitivity and flexibility of thermal methods often limits its use for this purpose. Therefore, the electrical measurement is widely used to measure AC loss. Main problem here is the capturing of correct loss voltage [1, 2]. It can be avoided when the signal from the cable terminations (containing large resistive part and therefore hindering sensitive measurement of tape properties) is used, suggesting that the difficulty stems in a non-uniform distribution of electrical field among the tapes in the cable. To clarify the question, we have constructed a short model and investigated experimentally the currents and voltages in individual tapes. We have found spread of values in the properties of tapes (critical currents, *n*-factors) as well as the contact resistances. Then, non-uniformity of the amplitudes and phases of currents in individual tapes was predicted by finite element calculations [3] and confirmed quantitatively by experiments. A simplified circuit model was developed to investigate, how the spread of current amplitudes and phases will be transferred into measured signals. Eventually we have found, that plausible AC loss results can be still achieved using some kind of averaging of the voltage signal taken from different places of the cable.

Manuscript received October 2, 2004. This work was supported in part by the Science and Technology Assistance Agency under contract APVT-20-012902 and by the European Commission (project ENK6-CT-2002-80658 ASTRA)

F. Gömöry, L. Frolek and J. Šouc are with the Institute of Electrical Engineering of the Slovak Academy of Sciences, Dubravska cesta 9, 84104 Bratislava, Slovakia (phone: 4212-5477-5820; fax: 4212-5477-5816; e-mail: fedor.gomory@savba.sk).

F. Grilli was with the Swiss Federal Institute of Technology - Lausanne, CH
1015 Lausanne, Switzerland. He is now with the Los Alamos National Laboratory, MST-STC, Los Alamos, New Mexico 87545, United States, e-mail fgrilli@lanl.gov.

II. Cable model and experiments

The cable model used for the verification of AC regime is the same as previously used to study the influence of non-uniformity on the DC critical current [4]. It consists of $N = 16$ straight parallel Bi-2223/Ag tapes with ~30 A of critical current fixed on a non-metallic cylindrical mandrel. Resulting DC critical current of the model reached 500 A in LN_2 bath. The cable termination was furnished with miniature Rogowski coils, to allow the determination of amplitude and phase of current in any tape with the help of a lock-in amplifier. The phase of lock-in was adjusted with respect to the total cable

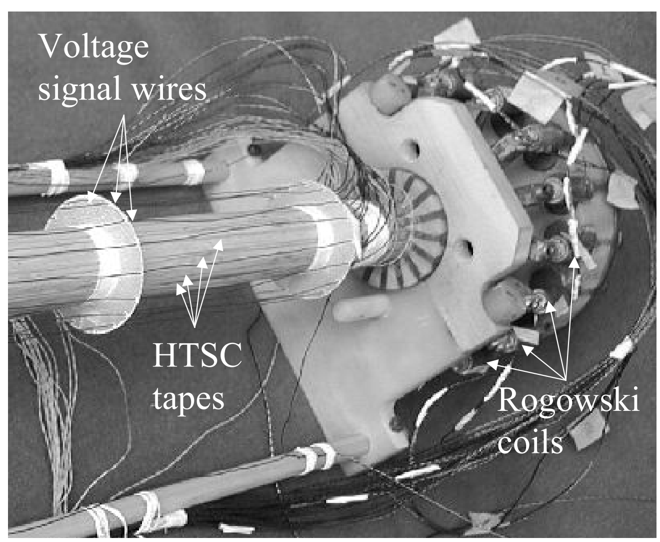

Fig. 1: Detailed view of the current termination of the cable model.

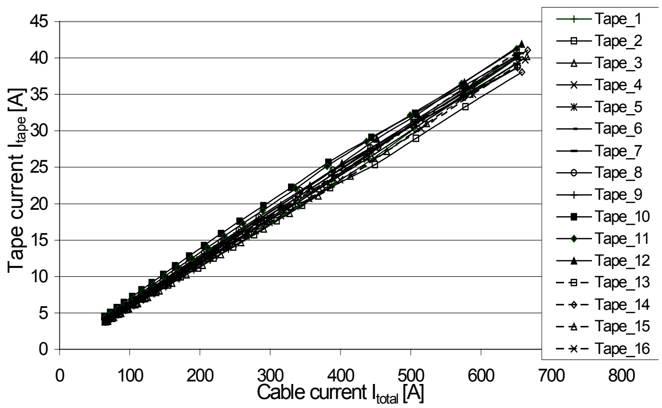

Fig. 2: Currents carried by individual tapes of the cable in AC regime (72 Hz, 77K). Differences in currents are mostly caused by the varying resistances of the soldered contacts between the tapes and the common current termination.

current I_{total}, measured by another Rogowski coil. The voltage signals have been led by wires lifted 10 mm over the tape's surface.

The data reported here have been taken at the frequency of 72 Hz, nevertheless similar results have been achieved also at other frequencies in the range 36 – 144 Hz. AC currents up to 650 A (r.m.s. value) have been supplied to the model.

First, the spread of tape properties, at the condition of supplying DC current to the whole cable, was established by the procedure described in [4]. The critical currents of tapes within the interval from 30.6 and 32.8 A have been fond, and the *n*-factors of the power-law current-voltage curve ranged between 22 and 24.5. We have also established the value of resistance, connected in series with each tape, after splitting the total cable current into parallel branches. The metallic parts of the current termination are identical for each branch, however the values of screw or soldered contacts could be quite irregular. Indeed, we have found the values of series resistances ranging from 74 to 106 μΩ. It is then not surprising, that the currents in tapes differ in amplitude, as shown in the Figure 2. Moreover, the phases of currents in tapes are no more equal, as demonstrate the experimental data plotted in Figure 3. The phase difference at low currents remain roughly constant until the amplitude of the cable current exceeds the DC critical current of the cable. Then the transition to a new distribution of phases takes place. In this work we concentrate on the normal operating conditions of the cable, i.e. below the critical current.

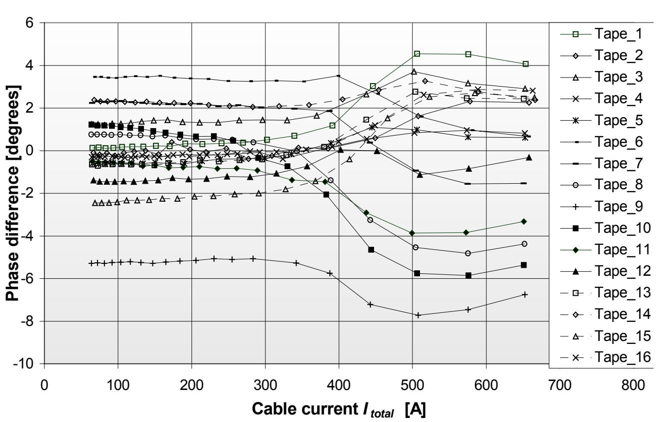

Fig. 3: Phase differences between the currents in tapes and the total cable current. Change of the dependence at the total currrent amplitude exceeding the critical current of the cable is remarkable.

III. Simulations of Current Distribution

To compare the experimental data with a theoretical prediction, FEM simulations using the software package FLUX3D [5] has been carried out. Each superconducting tape is replaced by parallel combination of a nonlinear media with power-law current-voltage curve, and a normal path with constant conductivity. Experimentally determined values of tape parameters and contact resistances has been inserted in the simulation, and current distribution and AC loss calculated [3]. Interestingly, the total AC loss is only weakly affected by the considered cable non-uniformities. The distribution of phases found by the FEM simulation was in nice agreement with experimental results, as illustrates the Figure 4. To understand better the underlying physical phenomena, we have developed a simplified analytical model, based on a circuit representation of the cable cross-section presented in the Figure 5. At low currents, the nonlinear resistance of a superconducting tape is negligible compared to the value of series resistance, thus the equivalent circuit representation becomes linear. The splitting of total cable current, I_{total}., in N =16 parallel tape currents, I_1, I_2,…I_{16}, is in AC conditions controlled by the impedances of parallel branches. In series to each resistance, a mutual inductance between the tape and the whole cable is placed. This is a way of representing the magnetic interaction between currents in the individual tapes. We have estimated that

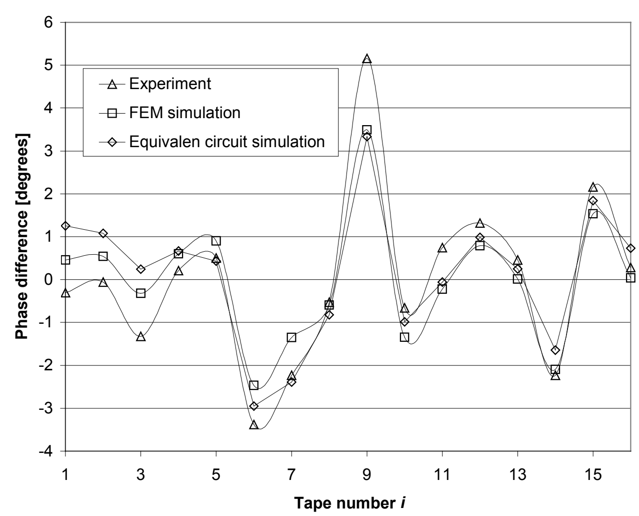

Fig. 4: Phase difference between the total current in the cable model and the current in the *i*-th tape. Experimental results at I_{total} = 200 A (triangles) are compared with theoretical predictions of FEM-based simulation (squares) and equivalent circuit calculations (diamonds).

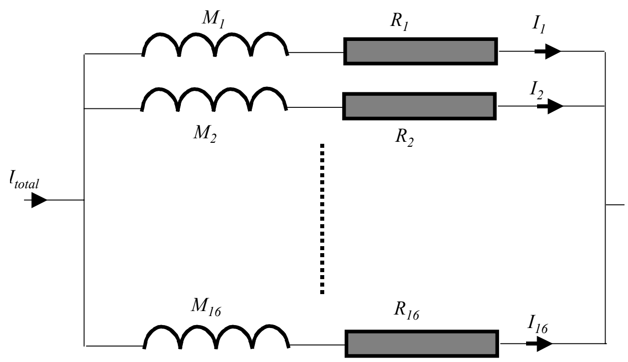

Fig. 5: Equivalent circuit for low currents. In each branch, the mutual inductance between the tape and the whole cable is placed in series with the resistance of metallic parts and ohmic contacts.

$$M_i = l\frac{\mu_0}{2\pi}\frac{R_c}{\sqrt{\frac{w_i.t_i}{\pi}}}$$

where l is the length of cable model, $\mu_0 = 4\pi.10^{-7}$ H/m, R_c is the mean radius on which the tapes are placed, and w_i, t_i, are the width and thickness, respectively, of the tape. It is not difficult to resolve the circuit from Figure 5, and show that the current in the i-th branch will be

$$I_i = I_{total}\frac{R_p + j\omega L_{cab}}{R_i + j\omega M_i} \quad (1)$$

where $j = \sqrt{-1}$, $\omega = 2\pi f$ is the angle frequency of AC current, $R_p = \left(\sum_i \frac{1}{R_i}\right)^{-1}$ and $L_{cab} = \left(\sum_i \frac{1}{M_i}\right)^{-1}$. The formula (1) can be reduced assuming that the tape dimensions do not vary considerably, thus all the mutual inductances are $M_i = M = l.2.10^{-7} R_c\sqrt{\pi/S_{tape}}$ where $S_{tape} = w.t$ is the cross-section of the tape. Inserting the values of our cable model, l = 0.97 m, R_c = 12.2 mm, w = 4.2 mm and t = 0.3 mm, we have found M = 0.58 μH. At f = 72 Hz, the ωM term will outweight the resistive part R_i by a factor of about three, suggesting further simplification of the expression (1) with very practical result:

$$I_i = \frac{I_{total}}{N}\left[1 + j\frac{R_i - NR_p}{\omega M}\right] \quad (2)$$

According to this prediction, one should expect flat distribution of current amplitudes, nevertheless with the phases of currents both positive and negative. Because $R_i << \omega M$, the phase shift of the i-th current could be approximated as

$$\varphi_i = \frac{R_i - NR_p}{\omega M}. \quad (3)$$

The results obtained using the formula (3) are also plotted in Figure 4, and agree surprisingly well with those determined with the help of complex FEM simulation.

IV. Influence on voltage signals

During the measurement of cable's AC loss, one should register the voltage in phase with the cable total current. In the case of tape currents mutually shifted in phase, local magnetic fields are generated that are not exactly in phase with the total cable current. Then, in the loss voltage measured by a pair of voltage taps, there will be also an additional false contribution due to these shifts. We have estimated this contribution by a computation based on the geometrical representation of the cable cross-section illustrated in the Figure 6. Total magnetic flux generated by all the tape currents in the loop formed by the k-th outer leg of signal wiring (at radius R_2) and voltage tap line (at radius R_1) is

$$\Phi_k = -l_t\frac{\mu_0}{4\pi}\left\{\sum_{i=1}^{N} I_i G_{k,i}\cos\varphi_i + j\sum_{i=1}^{N} I_i G_{k,i}\sin\varphi_i\right\} \quad (4)$$

where l_t is the distance between voltage taps and

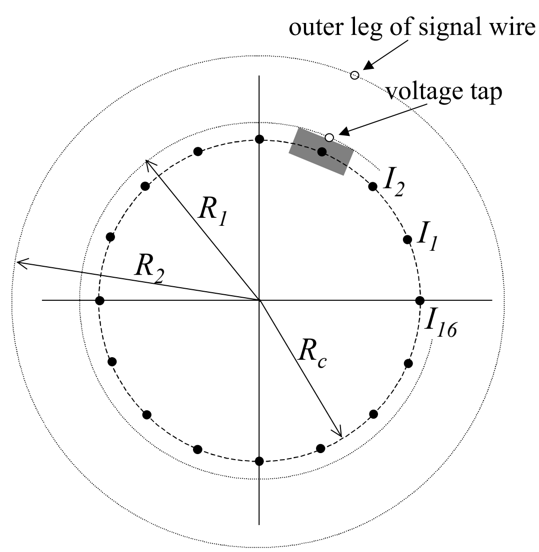

Fig. 6: Simplified cross-section of the cable used to estimate the false loss voltage signal. Tape currents are substituted by line currents in the tapes' centres. Just one tape with voltage tap and outer leg of signal wire, at the position k = 3 is shown in detail as a shaded area.

$$G_{k,i} = \ln \frac{(x2_k - x_i)^2 + (y2_k - y_i)^2}{(x1_k - x_i)^2 + (y1_k - y_i)^2}$$

with

$$x2_k = R_2 \cos\left(2\pi \frac{k}{N}\right); \quad y2_k = R_2 \sin\left(2\pi \frac{k}{N}\right)$$

$$x1_k = R_1 \cos\left(2\pi \frac{k}{N}\right); \quad y1_k = R_1 \sin\left(2\pi \frac{k}{N}\right)$$

$$x_i = R_c \cos\left(2\pi \frac{i}{N}\right); \quad y_i = R_c \sin\left(2\pi \frac{i}{N}\right)$$

The voltage induced by the flux (4) in the signal wire contacted to the k-th tape will be

$$U_k = l_t \omega \frac{\mu_0 I_{total}}{4\pi N} \left\{ -\sum_{i=1}^{N} G_{k,i} \sin\varphi_i + j \sum_{i=1}^{N} G_{k,i} \cos\varphi_i \right\} \qquad (5)$$

The second term in brackets is purely inductive, without any influence of the AC loss determination. However, the first term represents the voltage in phase with current, that would be evaluated as a loss voltage by the lock-in. To eliminate it in the experiment, we have found the following procedure: Let us take the real parts of voltage signals (5) from all the k = *1..N* = 16 tapes. We assume the same relative position, with respect to k-th tape, of k-th voltage taps and return leg of signal wire. The sum of these signals is

$$U_{sum} = -l_t \omega \frac{\mu_0 I_{total}}{4\pi N} \sum_{k=1}^{N} \sum_{i=1}^{N} G_{k,i} \sin\varphi_i \ .$$

One can reorder the summations to prove that

$$U_{sum} = -l_t \omega \frac{\mu_0}{4\pi} \sum_{i=1}^{k} \frac{I_{total}}{N} \sin\varphi_i \sum_{k=1}^{N} G_{k,i}$$

is indeed zero: $\sum_{k=1}^{N} G_{k,i} = G$ is a constant because of the summation over the complete circulation around the cable does not depend on the starting position. This constant could be moved before the summation of imaginary parts of the tape currents, that is now equal to zero because the total current is purely real.

Experimental confirmation of this idea is given in the Figure 7. The losses determined in traditional way, by direct multiplication of the in-phase signal measured on any of the tapes with the cable current are plotted by lines. Dashed part corresponds to negative measured voltage that is obviously charged by huge error. Taking the sum of measured voltages, false loss voltage contributions cancel out and the result is in good agreement with FEM calculation.

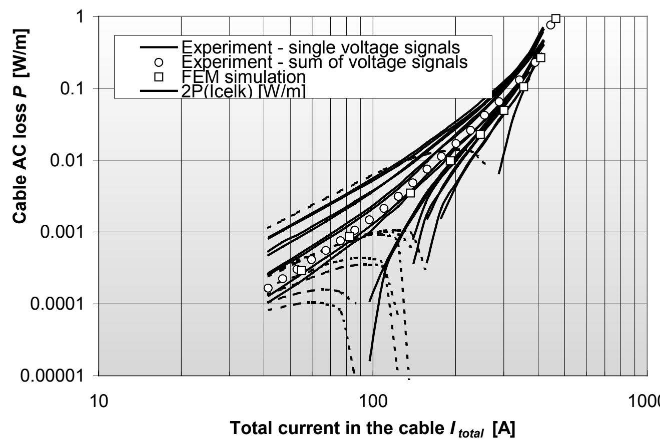

Fig.7: AC loss evaluation from the sum of measured voltage signals. Very good agreement with the prediction of the FEM calculation is achieved, in spite of huge scatter of the original voltage signals.

V. Conclusions

We have investigated the distribution of currents in a single layer superconducting cable. Phase shifts between currents lead to appearance of false loss voltage signals. This hinders the determination of transport AC loss by simple multiplication of the cable current by the voltage taken from a pair of taps placed in an arbitrary position in the cable. To find the correct value of AC loss, one has to sum voltages taken from a set of voltage signal wires distributed symmetricaly around the cable.

References

[1] S. K. Olsen, A. Kuhle, C. Traeholt, C. Rasmussen, O. Tonnesen, M. Daumling, C. N. Rasmussen, D. W. A. Willen, "Alternating current losses of a 10 metre long low loss superconducting cable conductor determined from phase sensitive measurements ," *Superconductor Science and Technology*, vol. 12, pp. 360-365, no. 6, June 1999.

[2] L. Gherardi, F. Gömöry, R. Mele and G. Coletta, "A.c. losses in Bi-2223 tapes for power applications," *Superconductor Science and Technology*, vol. 10, pp. 909-913, no. 6, June 1999.

[3] F. Grilli, S. Stavrev, B. Dutiot, S. Spreafico, R. Tebano, F. Gömöry, L. Frolek, J. Šouc, "Numerical modelling of a HTS cable in AC regime," *Physica C*, vol. 401, nos. 1-4, pp. 176 – 181, January 2004.

[4] F. Gömöry, L. Frolek, J. Šouc, G. Coletta, S. Spreafico, "Measurement of DC critical current in superconducting cable with non-uniformities," *IEEE Trans. Applied Superconductivity*, vol. 13, no.2, pp. 1968–1971, June 2000.

[5] CEDRAT SA-10, Chemin de Pre Carre-ZIRST 38246 Meylan, France. http://www.cedrat.com